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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

SECOND SEMESTER M.TECH DEGREE EXAMINATION, APRIL/MAY 2018

Electrical & Electronics Engineering

Control Systems, Guidance and Navigational Control

01EE6102 Optimal Control Theory

Answer *any two full* questions from *each* part

Limit answers to the required points.

Max. Marks: 60 Duration: 3 hours

PART A

1. a. Explain the steps involved in the mathematical formulation of an optimal control problem with a proper example. 5

   b. State and prove the fundamental theorem of calculus of variation 4

2. a. Determine the extremal for the functional $J(x)=\int_0^2 (\dot{x}^2+2x\dot{x}+4x^2)dt$ given that x(0) = 1, and x(2) is free 5

   b. Derive the necessary condition for a function to be an extremal for the functional $J(x)=\int_{t_0}^{t_f} g(x,\dot{x},t)dt$. In the (t,x) plane, the initial point (t_0,x(t_0)) is specified, final value of $x(t_f)$ is specified and the final time is free 4

3. a. Derive the necessary condition for a function to be an extremal for the functional $J(x)=\int_{t_0}^{t_f} g(x,\dot{x},t)dt$. In the (t,x) plane, the initial point (t_0,x(t_0)) is specified, final value of x may be constrained to lie on a specified moving point or curve $\theta(t)$ such that $x(t_f)=\theta(t_f)$ 4

   b. Determine the extremal for the functional $J(x)=\int_0^{t_f} \sqrt{(1+\dot{x}^2(t))}dt$ which has x (0) = 2 and terminates on the curve $\theta(t)=-4t+5$ 5

PART B

4. a. From the fundamentals, discuss, derive and comment on the statement, "An Optimal control must minimize the Hamiltonian" 4

b. Determine whether the problem of transferring the system 5

$$\dot{x}_1(t) = x_2(t)$$
$$\dot{x}_2(t) = u(t)$$

from any arbitrary initial state x_0 to a specified target set S(t) with perform

$$J = \int_0^{t_f} [\lambda + |u(t)| dt$$

has any singular interval. Final time is free

5. a. Distinguish between bang-bang control and bang-off-bang control 3

b. The system is $\dot{x}(t) = u(t)$ is to be transferred from an arbitrary initial state x_0 to the origin by minimizing the performance measure $J(u) = \int_0^{t_f} |u(t)| dt$ where t_f is free and the admissible controls satisfy $|u(t)| \le 1.0$. Determine the optimal control law 6

6. a. Explain singular intervals and derive the conditions for singular interval to happen in a time optimal problem 5

b. Find the set of reachable states for the system $\dot{x}(t) = u(t)$ where the admissible control must satisfy $-1 \le u(t) \le 1$ 4

PART C

7. a. Derive Hamilton-Jacobi-Bellman equation 5

b. The first order linear system $\dot{x}(t) = -10x(t) + u(t)$ is to be controlled to minimize the performance measure $J = \frac{1}{2}x^2(0.04) + \int_0^{0.04} \left[\frac{1}{4}x^2(t) + \frac{1}{2}u_2(t)\right] dt$. The admissible state and control values are not constrained by any boundaries. Find the optimal control law by using the Hamiltonian- Jacobi- Bellman equation 7

8. a. Explain the principle of optimality 2

b. A discrete system described by the difference equation $x(k+1) = x(k) + u(k)$ is to be controlled to minimize the performance measure $J = \sum_{k=1}^{2} \left[2|x(k) - 0.1k^2| + |u(k)|\right]$. The state and control values must satisfy the constraints $\begin{array}{l} 0 \le x(k) \le 0.4, k = 0,1,2 \\ -0.2 \le u(k) \le 0.2, k = 0,1 \end{array}$ 10

i. Use the dynamic programming algorithm to determine the optimal control law $u^*(x(k), k)$. Quantize the state into the values x(k)=0,0.1,0.2,0.3,0.4, k-=0,1,2 and the control into the values u(k)=-0.2,-0.1,0,0.1,0.2, (k=0,1).

ii. Determine the optimal control sequence $\{u^*(0), u^*(1)\}$ if the initial state value is x(0)-0.2

9. a. Derive the optimal control law for discrete linear regulator problem 6

b. Explain the imbedding principle 6